AP Calculus AB

WS 90 - Integral as Net Change

1) $s(t) = -t^3 + 7t^2 - 14t + 8$ m

a) $v(t) = -3t^2 + 14t - 14$

b) $v(4) = -3(16) + 14(4) - 14 = -6$ m/sec

Since $v(4) < 0$, the particle is moving to the left.

c) $ROC_{Avg} = \frac{s(12) - s(0)}{12 - 0}$

$= -74$ m/sec

d) $v(t) = 0$

$-3t^2 + 14t - 14 = 0$

$3t^2 - 14t + 14 = 0$

~~$(3t \quad)(t \quad) = 0$~~

$t = \frac{14 \pm \sqrt{(-14)^2 - 4(3)(14)}}{6}$

$= 1.451, 3.215$

2) $Q(t) = 200(3-t)^2 \rightarrow$ # of gallons of H_2O

a) $Q'(t) = 400(3-t) \cdot (-1)$

$Q'(10) = 400(-7) \cdot (-1)$

$= 2800$ gal/min

H_2O is running out of the tank @ a rate of 2800 gal/min

b) $ROC_{Avg} = \frac{\Delta Q}{\Delta t} = \frac{Q(10) - Q(0)}{10}$

$= \frac{200(-7)^2 - 200(3)^2}{10}$ gal/min

$= \frac{40(200)}{10} = 800$ gal/min

3) $W(0) = 81.637$

$W'(t) = 9\sin(\sqrt{t+1})$

a) $W'(4) = 9\sin\sqrt{5} = 7.081$ gal/min

b) $W(t) = 81.637 - \int_0^t 9\sin(\sqrt{x+1})\,dx$

c) $W(6) = 81.637 - \int_0^6 9\sin(\sqrt{x+1})\,dx$

$= 36.606$ gallons

At the end of 6 minutes, the tank contains 36.606 gallons of H_2O.

4) $R(t) = 2\sqrt{t + 5t^3}$ → rate at which H_2O is sprayed

a) $W(t) = \int_0^t R(x)\,dx$

b) $W(10) = \int_0^{10} R(x)\,dx = 568.109$ gallons

In the first 10 minutes, 568.109 gallons has been sprayed onto the field of vegetables.

5) $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ → rate of enter

$L(t) = 645$ → rate of removal

a) $E(3) = 135$ gal/hr

b) $\int_0^4 E(t)\,dt = 3981.022$ gallons

c) Total gallons = Starting quantity + Quantity Enter − Quantity Removed

$$= 0 + \int_0^3 E(t)\,dt - \int_0^3 645\,dt$$

$$= 1417.193 \text{ gallons}$$